Constructive Jűrg

A Personal Overview of Constructive Quantum Field Theory

by Arthur Jaffe

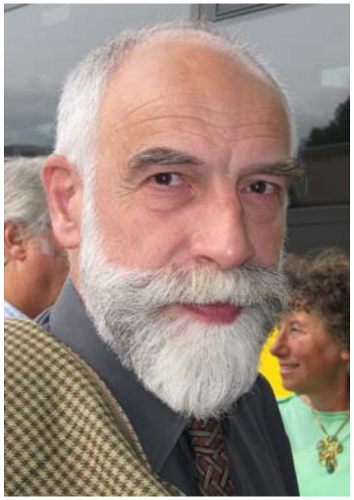
Dedicated to Jürg Fröhlich

3 July 2007

E.T.H. Zürich

“Ich heisse Ernst, aber ich bin fröhlich;
er heisst Fröhlich, aber er is ernst!”
Richard Ernst

1977 @ 31



29 June 2007 @ 61 - ϵ

HAPPY Birthday!!!

Jürg: over 285 publications with over 114 co-authors

Abou Salem	Walid
Aizenman	Michael
Albanese	Claudio
Albert	Carlo
Alekseev	Anton Yu.
Ambjorn	J.
Aragao de Carvalho	C.
Aschbacher	W.H.
Bach	Volker
Bellissard	Jean
Benettin	G.-C.
Birke	L.
Borgs	Christian
Bovier	Anton
Bricmont	Jean
Brydges	David
Bugliaro Goggia	Luca
Caracciolo	S.
Cattaneo	A.S.
Chamseddine	Ali H.
Chayes	Jennifer
Chayes	Lincoln
Cheianov	V.
Chen	Thomas
Constantinescu	C.
Cotta-Ramusino	Paolo
Datta	N.
Driessler	W.
Durhuus	B.
Eckmann	Jean-Pierre
El Mellouki	A.
Epstein	Henri
Faddeev	Ludwig
Fefferman	Charles
Felder	Giovanni
Fernandez	R.
Ferrari	Lucio
Fisher	Daniel
Fuchs	Juergen
Gabbiani	F.
Gawedzki	K.
Gestafsson	S.
Gidas	B.
Giffen	C.
Giorgilli	A.
Glaus	U.
Goetschmann	Roland
Graf	G.-M/
Graffi	S.
Grandjean	Olivier
Griesemer	M.
Gustafson	Steven
Hasler	D.
Hoppe	Jens
Huckaby	Dale
Imbrie	John
Israel	Robert
Jonsson	B.L.G.
Jonsson	Thordur
Keller	G.
Kerler	Thomas
King	Christopher
Lenzmann	E.
Leupp	M.
Lieb	Elliott
Loss	Michael
Marchetti	P.A.
Mardin	A.
Martinelli	F.
Merkli	Marco
Morchio	G.
Orland	P.
Osterwalder	Konrad
Park	Young-Moon
Pedrini	Bill
Pfeifer	P.
Pfister	Charles
Pizzo	Alessandro
Recknagel	A.
Rey-Bellet	Luca
Richard	J.-M.
Rivasseau	Vincent
Ruelle	David
Runkel	Ingo
Russo	L.
Schlein	Benjamin
Schnee	K.
Schomerus	Volker
Schwarz	S.
Schweigert	Christoph
Scoppola	E.
Seifert	M.
Seiler	Erhard
Sigal	Israel
Simon	Barry
Soffer	Avi
Sokal	Alan
Spencer	Thomas
Strocchi	Franco
Struwe	Michael
Stubbe	J.
Studer	U.M.
Thiran	E.
Troyer	M.
Tsai	T.-P.
Ueltschi	D.
van Enter	A.C.D.
Walcher	J.
Wayne	Clarence Eugene
Wittwer	Peter
Yau	H.-T.
Yau	S.-T.
Zee	Anthony
Zegarlinski	B.

Quantum Field Theory

Two major pedestals of 20th century physics:

Quantum Theory and Special Relativity

Are they compatible?

Motivated by Maxwell, Dirac

Quantum Electrodynamics (QED): light interacting with matter.

Rules of Feynman ⇒ Lamb shift in hydrogen and magnetic moment μ of the electron.

calculation: Bethe, Weisskopf, Schwinger, Tomonaga, Kinoshita,….

measurement: Kusch,…, Dehmelt, Gabrielse (1947-2007) now 60!

$$\mu = \kappa \frac{e\hbar}{2mc}\ , \qquad \kappa_{\mathrm{Bohr}} = \frac{1}{2}\ , \qquad \kappa_{\mathrm{Dirac}} = 1\ .$$

$$\kappa_{60} = 1.001\ 159\ 652\ 180\ 85(\pm 76).$$

Agreement of calculation with experiment: 1 part in 10^{12}

Strange, but Apparently True

Classical mechanics, classical gravitation, classical Maxwell theory, fluid mechanics, non-relativistic quantum theory, statistical mechanics,…, all have a logical foundation.

They all are branches of *mathematics*.

But: Most physicists believe that QED on its own is mathematically inconsistent.

Reason: It is not "asymptotically free" (1973).

Physical explanation: "need other forces."

Axiomatic Quantum Field Theory

Effort begun in the 1950's to give a mathematical framework for quantum field theory and special relativity.

Wightman, Jost, Haag

Lehmann, Symanzik, Zimmermann

Constructive Quantum Field Theory

Attempt begun in the 1960's: find examples (within this framework) having non-trivial interaction.

1. Construct non-linear examples.
2. Find the symmetries of the vacuum, spectrum of particles and bound states (mass eigenvalues), compute scattering, etc.
3. Work exactly (*non-perturbatively),* although with motivation from perturbation theory.

From Dream to Reality

- 1960 Mathematical theory appeared beyond reach
- 1973 dimension $d = 2, 3$ shown to work
- 2007 Still unresolved: $d = 4$.

ETH-Link

1961 Hercegnovi (Yugoslavia) summer school.

I went from Cambridge to hear lectures of Kurt Symanzik. Met fellow student, Klaus Hepp.

Jost/Wightman seminar took place during the fall at IAS, as I started to study there.

1963 Klaus Hepp came with Res Jost to Paris, where I spent a year with Arthur Wightman.

1964 Klaus & Marie-Claude came to Princeton.

Princeton 1964

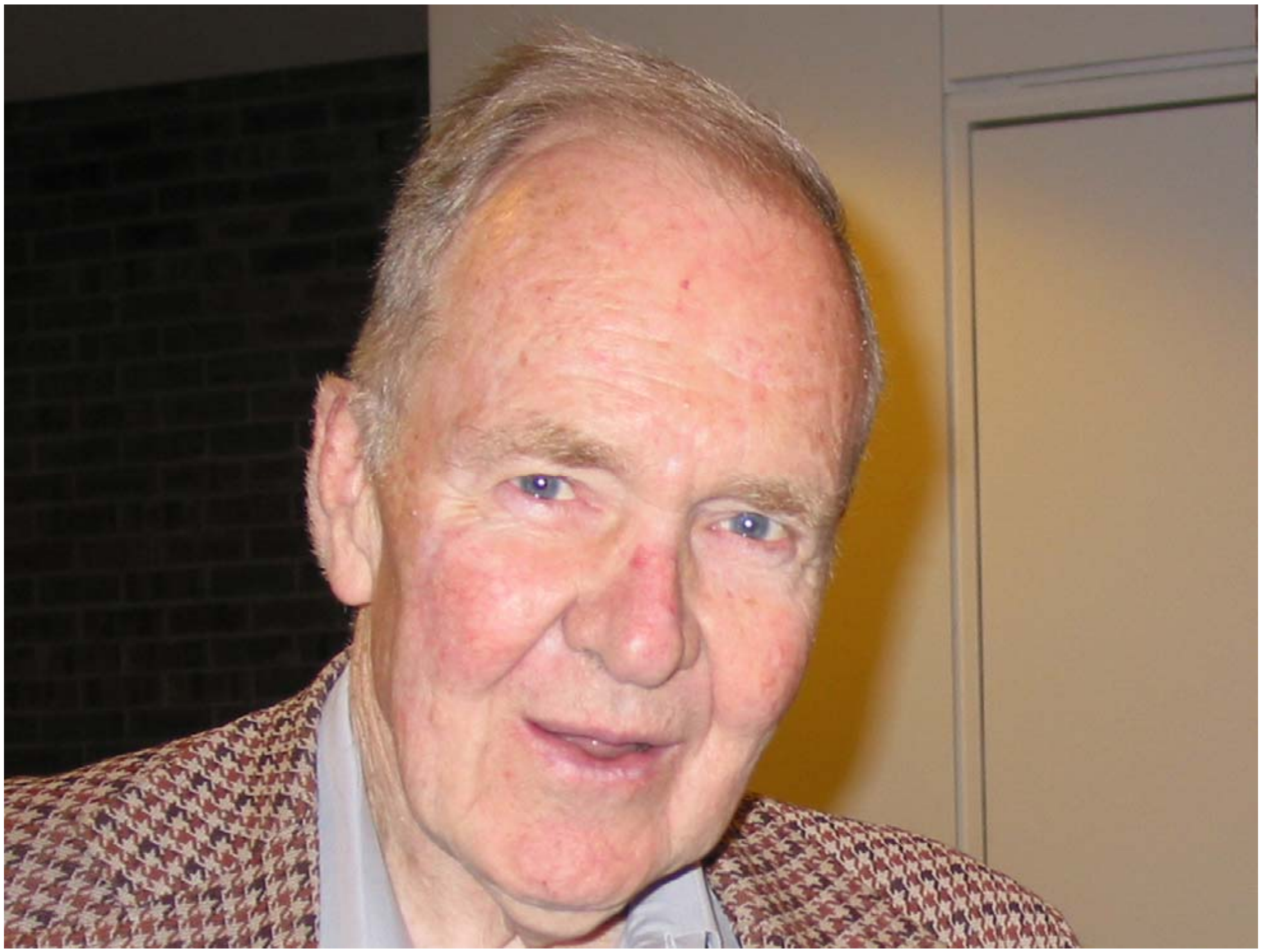
Klaus Hepp

Arthur

Princeton 1964

Res Jost: there in spirit!   Edward Nelson   Henri Epstein   Arthur Wightman

Arthur Wightman



Proposal: Jost, Wightman, Segal

Get non-perturbative solution for a non-linear field, e.g.

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\varphi(x) + \varphi^3(x) = 0 \ ,$$

where $\pi = \partial\varphi/\partial t$,satisfies the quantum constraint

$$[\varphi(\vec{x}, t), \pi(\vec{y}, t)] = i\delta\left(\vec{x} - \vec{y}\right) \ .$$

Problem in solving such a non-linear equation:

The non-linearity $\varphi(x)^3$ does not exist.

Requires “renormalization.”

Case d = 2,3 $\quad \varphi(x)^3 \to \varphi(x)^3 - \alpha\varphi(x) \ .$ in equation

$\varphi(x)^4 \to \varphi(x)^4 - 2\alpha\varphi(x)^2 + \beta \ .$ in energy density

Two Possible Approaches

1. Study Canonical Quantization

   Hamiltonian H, field φ act as operators

$$0 \leq H = H^* \ , \qquad \varphi(t) = e^{itH} \varphi e^{-itH}$$

Jost, Wightman, Segal

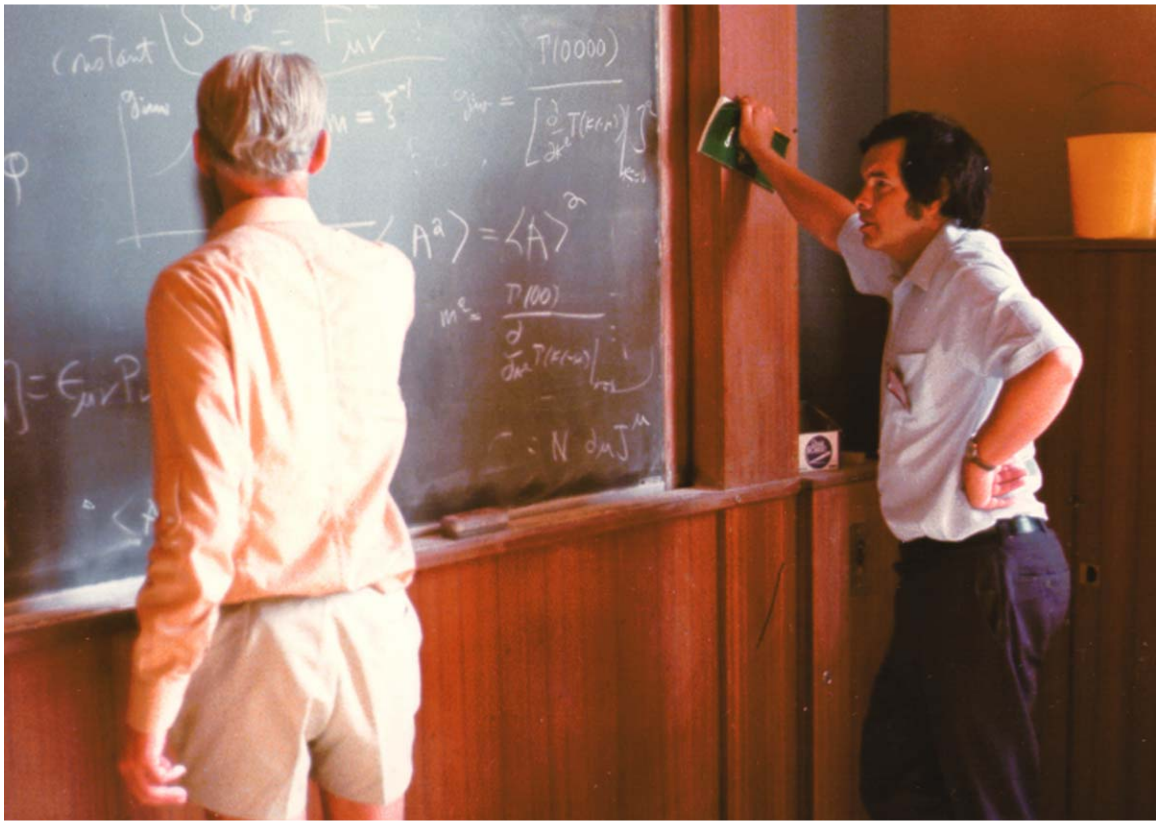
2. Study Functional Integral

   "Euclidean" action *S*;   Classical Stat. Mech.

$$d\mu(\Phi) = \frac{1}{Z} e^{-S(\Phi)} d\Phi$$

Symanzik

Kurt Symanzik



Kurt Symanzik　　　　Ken Wilson

Which method is better?

Much debate in the 1960's.

Debate wrong. In the end, the two methods complement one-another. When taken together they yield strong results.

$$\left\langle A\Omega, e^{-tH} B\Omega \right\rangle_{\mathcal{H}} = \int_{\mathcal{S}'} \overline{A}\, B(t) d\mu(\Phi) \ .$$

Canonical

Classical S.M.

1968: First non-trivial interaction

Self-adjoint, finite-volume H=H* for $\varphi^4{}_2$ equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\varphi(x) + \varphi^3(x) = 0\ .$$

Jim Glimm Arthur (photo 1979)

Philosophy: Use “Estimates”

first order estimate:

$$\epsilon H_0 \leq H + M$$

second order estimate:

$$\epsilon H_0^2 \leq H^2 + M$$

field bound (volume-independent):

$$\pm\varphi(f) \leq \|f\| \, (H + I)$$

1968: First Visit to E.T.H.
Seminar für Theoretische Physik

Hochstrasse 60

Zürich

1969: Res Jost: “Honorary Member of the Seminar für Theoretische Physik”

My Course had Good Students

Robert Schrader

PhD student finishing with Klaus Hepp

Konrad Osterwalder

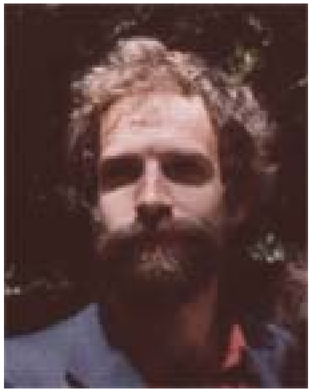
beginning PhD student and my Assistant

"There is an undergraduate student who writes very *long* and *complete* solutions to the exercises." (I only met him the next year, because….)

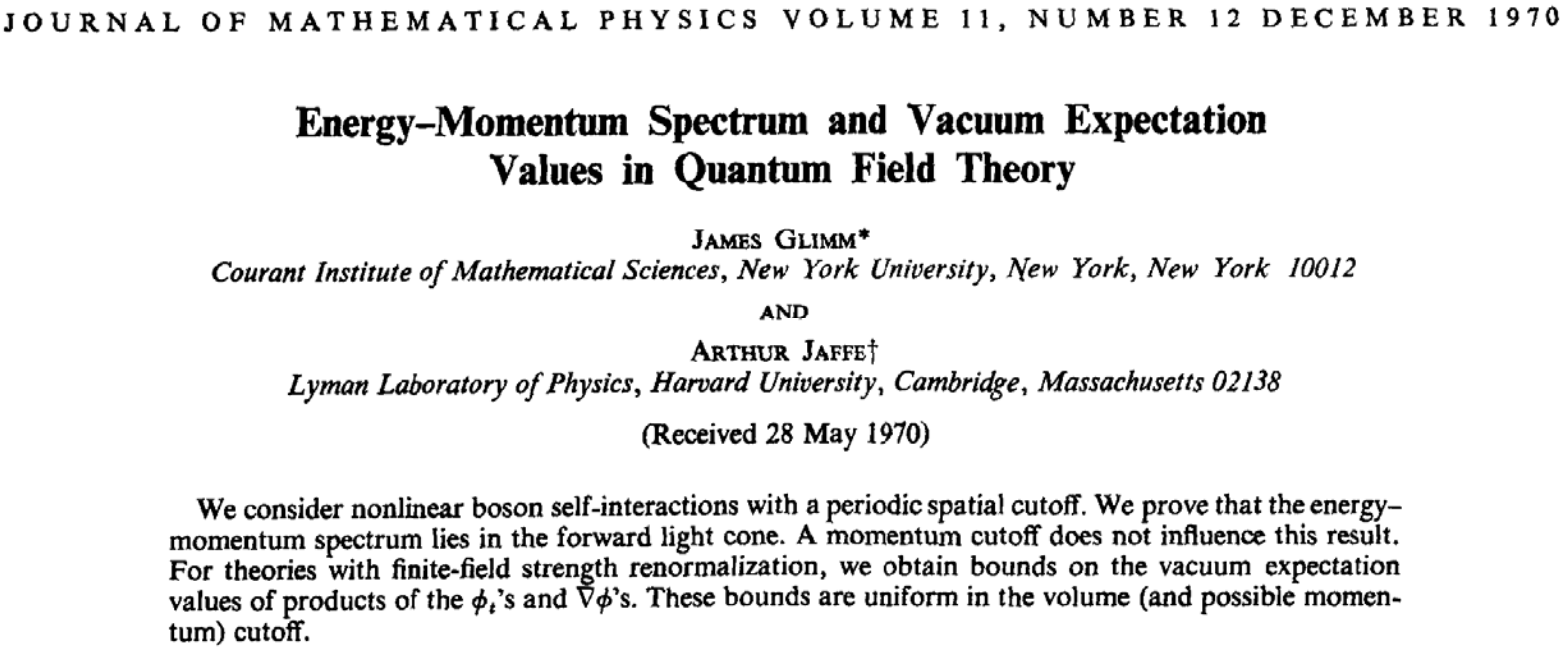
Today he writes many long solutions, both with success in mathematical physics,

as well as for the E.T.H. Schulleitung, ...

Les Houches 1970



? XX !!!!! ??



JOURNAL OF MATHEMATICAL PHYSICS VOLUME 11, NUMBER 12 DECEMBER 1970

Energy-Momentum Spectrum and Vacuum Expectation Values in Quantum Field Theory

JAMES GLIMM*
Courant Institute of Mathematical Sciences, New York University, New York, New York 10012
AND
ARTHUR JAFFE†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 28 May 1970)

We consider nonlinear boson self-interactions with a periodic spatial cutoff. We prove that the energy–momentum spectrum lies in the forward light cone. A momentum cutoff does not influence this result. For theories with finite-field strength renormalization, we obtain bounds on the vacuum expectation values of products of the ϕ_t's and $\nabla\phi$'s. These bounds are uniform in the volume (and possible momentum) cutoff.

Correction

Taking the logarithm, dividing by T and letting $T \to \infty$, the left side converges to $-E(\sim X_a)$. This gives the contradiction $0 \leq -\tau^{-1}$, and proves (4.1).

5. A CORRECTION

Our proof[5] of the spectrum condition $P^2 \leq H^2$ contains a gap, as was pointed out by Fröhlich and Faris. Namely, we required the Lorentz rotated Hamiltonian in a periodic box to have a simple ground state, which does not follow, as claimed, from standard methods. The remaining results in Ref. 5 are either independent of this gap or are proved (and improved) by the present paper, with the exception of the estimate

$$\pm \nabla\varphi(h) \leq \text{const}\|h\|_2(H + I).$$

This bound is Theorem 1.1, Case 2, $\epsilon = \frac{1}{2}$, and it should follow from the methods of the present paper.

[5] J. Glimm and A. Jaffe, J. Math. Phys. **11**, 3335 (1970).

Same Error!

Erratum to "Construction of non-linear local quantum processes: I"

By Irving Segal

Annals of Mathematics, Vol. **92** (1970), 462-481 (noted by A. Klein)

In the statement of Theorem 1, insert "real" before "operator".

Thesis

1. On the Infrared Problem in a Model of Scalar Electrons and Massless Scalar Bosons, Ph.D. Thesis, ETH 1972, published in Annales de l'Inst. Henri Poincaré, *19*, 1-103 (1974).

MR0368649 (51 #4890) 81.46

Fröhlich, Jürg

On the infrared problem in a model of scalar electrons and massless, scalar bosons. (French summary)

Ann. Inst. H. Poincaré Sect. A (N.S.) **19** (1973), 1–103.

In this interesting and comprehensive article, the infrared (IR) problem is investigated in the framework of certain simple models. These models discuss the interaction of conserved, charged, scalar particles and relativistic, neutral, scalar bosons of zero rest mass.

A renormalized Hilbert space is constructed such that the spectrum of the resulting energy-momentum operator contains a unique one-electron shell corresponding to dressed one-electron space (DES). Several concepts for the collision theory on the charge-one sector are developed, with and without IR cutoff.

A list of unsolved problems in the field adds to the value of the work.

Reviewed by *P. Achuthan*

first postdoctoral fellows

Robert Schrader

1970-73

Konrad Osterwalder

1971-77

Jürg Fröhlich

1973-4

1968-1973

- Infinite Volume Theories, d=2. (first non-linear examples of the Wightman axioms)
- Yukawa-like interactions with fermions.
- Euclidean-Invariant Functional Integral Methods
- Euclidean Axioms ≡ Wightman Axioms
- Finite volume $\lambda\varphi^4$ in dimension 3.

Infinite Volume Interaction

- Compactness Method (Glimm-J.; Schrader)

  Based on $-E_0 \leq |\mathrm{Vol}|$ and C*-algebra methods.

  Convergence of subsequences: Ω not invariant
- Cluster Expansions (Glimm-J.-Spencer)

  Convergence. Also isolated one-particle states (upper and lower mass gap) for small coupling constants. First example of axioms!!
- Correlation Inequalities (Guerra-Rosen-Simon)

  Special bosonic interactions

Euclidean-Invariant Methods

- Symanzik: formulated Euclidean, Markov field program
- Nelson: positivity of $\lambda\varphi^4_2$ and mathematical formulation of Euclidean Markov field
- Guerra: Easy proof of vacuum energy density bound
- Glimm-J.: positivity of renormalized $\lambda\varphi^4_3$
- Guerra, Rosen, Simon: infinite volume limit
- G.-J.-S.: infinite volume limit and particles

Euclidean ↔ real time

Nelson Markov field

Osterwalder-Schrader Axioms (general)

OS Assumptions on Euclidean Green's Functions S_n

1. Regularity Condition
2. Euclidean Covariance
3. Reflection (Osterwalder-Schrader)-Positivity:

$\theta(\vec{x}, t) = (\vec{x}, -t), \text{ and } t_j < t_{j+1},$

$$0 \leq S_{2n}(\theta x_n, \ldots, \theta x_1, x_1, \ldots, x_n)$$

4. Clustering

OS Theorem

OS 1,2,3 ↔ Wightman axioms+/- (regularity, non-unique Ω)

OS 1,2,3,4 ↔ Wightman axioms+ (regularity, unique Ω)

Higher spin Ozkaynak, Lattice Gauge Fields Osterwalder-E. Seiler
Super-symmetry generalization Osterwalder

Massachusetts

Snow in Watertown, 1973
Osterwalders

1977
Fröhlichs

Jürg's Generating Functional Axioms

$$S(f) = \int e^{i\Phi(f)}\, d\mu(\Phi)$$

Invariance, Regularity, and Positivity in terms of S, the Fourier tranform of the measure μ. For example, RP becomes

$$0 \leq \sum_{i,j=1}^{n} \overline{c_i} c_j S(\theta f_i - f_j)\ ,$$

for $\quad \mathrm{suppt.} f \in R_+^d\ .$

Helvetica Physica Acta
Vol. 47, 1974

Birkhäuser Verlag Basel

Schwinger Functions and their Generating Functionals, I

by Jürg Fröhlich[1])
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

(25. III. 74)

Abstract. (Euclidean) Markoff fields and in particular the fields of the $P(\varphi)_2$ models in two-dimensional space-time are studied. It is shown that the states on the Markoff fields, i.e. the generating functionals for the Schwinger functions of the $P(\varphi)_2$ models with different boundary conditions, converge as the interaction region tends to $\mathbb{R}^2$. The generating functionals in the infinite volume limit are in 1–1 correspondence with Euclidean invariant measures on $\mathscr{S}'$; (here $\mathscr{S} = \mathscr{S}_{real}(\mathbb{R}^2)$). Existence of coincident Schwinger functions and continuity in the time arguments are proven. The Wightman axioms are verified for the quantum fields in the infinite volume limit. Some results on the general structure of Markoff field theory are presented.

Ann. Inst. Henri Poincaré,
Vol. XXI, n° 4, 1974, p. 271-317.

Section A :
Physique théorique.

Verification of Axioms for Euclidean and Relativistic Fields and Haag's Theorem in a Class of $P(\varphi)_2$-Models (*)

by

Jürg FRÖHLICH (**)
Lyman Laboratory of Physics
Harvard University
Cambridge, Massachusetts 02138

ABSTRACT. — Axioms for Euclidean (Bose) Fields are proposed and shown to suffice for the reconstruction of Relativistic Quantum Fields satisfying the Wightman Axioms. Those axioms are verified for a class of $P(\varphi)_2$ models. They seem to provide a suitable framework for (Bose) Field Theories in Two and three space-time dimensions. It is shown that the $P(\varphi)_2$-interacting field in the infinite volume limit is not a (generalized) free field or a wick polynomial of a (generalized) free field. If P_1 and P_2 are two interaction polynomials in the region of convergence of the Glimm-Jaffe-Spencer Cluster Expansion then the corresponding infinite volume field theories are different, unless $P_1(\xi) = P_2(\pm \xi + a) + b$.

ADVANCES IN MATHEMATICS 23, 119–180 (1977)

Schwinger Functions and Their Generating Functionals. II. Markovian and Generalized Path Space Measures on $\mathscr{S}'$

JÜRG FRÖHLICH*

Department of Physics, Harvard University, Cambridge, Massachusetts 02138

A general theory of $\mathscr{S}$-quasi-invariant, Markovian, and generalized path space measures on the Schwartz distribution space $\mathscr{S}'$ is developed. Ergodicity properties of such measures under the action of different transformation groups of $\mathscr{S}'$ of interest to quantum field theory are studied. As a result an algebraic approach to Markov field theory and a general, probabilistic (Euclidean) framework for the study of spontaneous symmetry breaking and phase transitions in quantum field models involving fundamental scalar fields such as the ϕ^4-model (or the generalized Yukawa model in two space–time dimensions) are presented. Applications of these results to the $P(\phi)_2$ quantum field models are given and it is rigorously proven that the short-distance behavior of these models in each pure phase is canonical and the long-distance behavior is determined by the physical mass.

3 papers, 150 pages

Mathematical Reviews

MR0436830 (55 #9768a) 81.46
Fröhlich, Jürg
Schwinger functions and their generating functionals. I.
Helv. Phys. Acta **47** (1974), 265–306.

MR0436831 (55 #9768b) 81.60
Fröhlich, Jürg
Schwinger functions and their generating functionals. II. Markovian and generalized path space measures on S^1.
Advances in Math. **23** (1977), *no. 2*, 119–180.

This work deals with a two-dimensional quantum field theory which can be described in terms of the Schwinger functions (Euclidean Green functions) composed by the tempered, local Bose quantum field on $\mathbf{R}^2$. The framework is highly mathematical and general, but by the simultaneous imposition of physical positivity and the Euclidean invariance one cannot get rid of the restriction of the "two dimensions".

The author's starting insight is in observing the correspondence between the quantum field theory and the Markov field theory, and he considers that the use of the latter is pertinent to the study of the $P(\varphi)_2$ models. Let J be a characteristic functional on the space $\mathcal{S}$ subject to certain properties of the Markov field theory. Then, J may be expressed as a Fourier transform of a probability measure ν defined on a σ-ring generated by the cylinder Borel subset of $\mathcal{S}'$. The Schwinger functions then correspond to the moments of the measure and, hence, J is the generating functional of the Schwinger functions. One of the author's purposes is, therefore, to construct an invariant Euclidean measure ν on $\mathcal{S}'$ in such a way that its moments, i.e., the Schwinger functions, are the Schwinger functions of the $P(\varphi)_2$ model and obey the Osterwalder-Schrader axioms. Roughly speaking, Part I deals with the functional J and Part II treats exclusively the invariant measures. {The readers are recommended to first of all read the first section of Part II, where the main results of Part I are summarized in Theorem I and the results of Part II in Theorems IIA and IIB. The readers will also find in this section an "introduction for mathematical physicists" as well as an "introduction for mathematicians".}

In the first half of Section 3 of Part I, general properties of J, in particular its continuity and analyticity, are discussed in detail, and it is shown that the Schwinger functions, derived from J, are tempered and positive in the sense of Osterwalder and Schrader. In the second half, the general results are applied to the characteristic functional for the cutoff $P(\varphi)_2$ theory, that is, to the functional of the Schwinger functions whose space-time is cut off in a finite region. The connection between the cutoff Hamiltonian theory and the Schwinger function theory is given in terms of the Feynman-Kac-Nelson path-integral formula. Section 4 of Part I is concerned with the infinite volume limit for J and asymptotic Euclidean invariance of the Schwinger functions. The existence of quantum fields in the infinite volume limit is also proved. Finally, under the proviso $P(\varphi)=\sum a_m\varphi^{2m}+\mu\varphi(\mu\geq 0)$, the author verifies the axioms proposed by Osterwalder and Schrader for $P(\varphi)_2$ model.

In Part II the author uses in an essential way a measure, whose characteristic functional is equal to the generating functional of the Schwinger functions, defined as the generalized path-space measures, or "path-space measures of processes that can be viewed as natural generalizations of some class of Markov processes and the transition function of which uniquely determines the quantum dynamics". To be able to consider the distributions $\{\mathfrak{S}_n\}_{n=0}^{\infty}$ as the moments of the probability measures (hence, Symanzik-Nelson positive) means to afford a powerful tool not only for the construction of the Bose quantum fields but also for the analysis of the phase transitions: the author, furthermore, claims that the results obtained are also relevant to the case of equilibrium states at positive temperature in view of the correspondence between the vacuum and the equilibrium states.

The first part of Part II is devoted to the description of the mathematical properties of ν (Section 2), for example, ergodic and local properties, equivalence classes of measures and so on. In particular, the last topic of Section 2 is the analysis of the quasi-invariant measure under a general transformation group $\mathfrak{G}$ of $\mathcal{S}'$ and of σ-algebra generated by the Borel cylinder sets of $\mathcal{S}'$. The author then proves stability theorems for the decomposition of ν into $\mathfrak{G}$-ergodic components. (Remarks on p. 147 instructively provide some comments on the physical significance of theorems.) Section 3 of Part II digresses to the free Euclidean case and shows how the general theory of measures fits to the Gaussian measure on $\mathcal{S}'$. In the final section an application of the preceding results to the $P(\varphi)_2$ quantum field models is given. It contains a list of locally correct interacting $P(\varphi)_2$ measures, from which the author constructs the Euclidean invariant infinite-volume interacting $P(\varphi)_2$ measure, that is a non-Gaussian example. Its general properties, including the "time"-translation ergodicity of measures, are also described. (As a corollary of a theorem, it is proved that the infinite-volume $P(\varphi)_2$ quantum field is not in the Borchers class of the free fields.) A comment on the problem of the non-ergodicity under the time-translation groups, i.e., the spontaneous symmetry breaking, then follows.

The "Note added in proof" indicates the new developments in the $P(\varphi)_2$ model theory since the time when the original of Part II was submitted (in summer of 1974).

Reviewed by *Masatsugu Minami*

MR0373494 (51 #9694) 81.46
Fröhlich, Jürg
Verification of axioms for Euclidean and relativistic fields and Haag's theorem in a class of $P(\varphi)_2$-models.
Ann. Inst. H. Poincaré Sect. A (N.S.) **21** (1974), 271–317.

The author proposes axioms for Euclidean (Bose) fields, formulated in terms of the generating functional (Fourier-Laplace transform) $J(f)=\int e^{i\varphi(f)}\,d\varphi$ of the Euclidean measure $d\varphi$. The axioms are shown to be sufficient for the reconstruction of relativistic quantum fields satisfying the Wightman axioms. The axioms are verified for even $P(\varphi)_2$ quantum fields with half-Dirichlet boundary conditions; they should be valid for all two- and three-dimensional (superrenormalizable) fields, and presumably renormalizable four-dimensional fields as well. The $P(\varphi)_2$ fields constructed by the author are shown to differ from free, generalized free, and Wick powers of generalized free fields. The author's development of the method of generating functionals has been useful for a variety of problems because it leads efficiently from bounds on the vacuum energy to bounds on Schwinger functions.

Reviewed by *J. Glimm*

Free-Field RP: S_2 = C= $(-\Delta + 1)^{-1}$

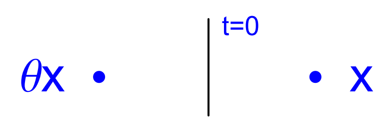
- RP reduces to Fröhlich

$$0 \leq \theta C|_{L^2(R^{d-1} \times R_+)} \; .$$

- Monotonicity in Boundary Conditions Glimm, J. (1979)

$$\mathrm{RP} \leftrightarrow C_D \leq C_N \; , \quad \text{on } t = 0 \; .$$

$$C - \theta C \leq C + \theta C \; .$$



- Static space-time generalization, $R^{d-1} \rightarrow M$

De Angelis, De Falco, Di Genova (1986); J., Ritter (2007)

$$0 \leq \theta C|_{L^2(M \times R_+)} \; .$$

Some Nice Jürg-Things

T > 0 Reconstruction Theorem

Analysis of Euclidean Borchers algebra and reconstruction of local observables Fröhlich (1975), Driesller (1977)

Pure Phases

Decomposition of $d\mu$ into ergodic components Fröhlich (1975), Fröhlich-Simon (1977)

Solitons

Soliton sectors in Sine-Gordon and φ^4 models Fröhlich (1976)

Soliton mass and surface tension Fröhlich-Bellisard-Gidas (1978)

Massive Thirring Model

Existence and Particle structure
Fröhlich-E.Seiler (1976)

$\mathrm{Exp}(\varphi)_2$ Model

Fröhlich-Park (1977)

Asymptotic Perturbation Theory

Perturbation theory yields correct S-Matrix in known models
J.-P. Eckmann-Epstein-Fröhlich (1976)

Lattice Y-M and Random Surfaces

Fröhlich-Park (1977)

Abelian Gauge-P$(\varphi)_2$ Models

Finite volume, continuum limit
Brydges-Fröhlich-E. Seiler (1979-1981)

Mass Gap (Higgs Effect), Infinite Volume Limit. Method: An Inductively defined renormalization expansion to yield true infra-red behavior
Balaban, Brydges, Imbrie, J. (1984-1985)

Theory of Symmetric Semigroups

Unbounded Semigroups $e^{\chi M}$. Examples arise from quantization of Euclidean rotations. They are putative analytic continuation of unitary $e^{i\chi M}$
Fröhlich (1981) and Klein-Landau (1981)
Fröhlich-Osterwalder-Seiler (1983) "Virtual Representations"
J.-Ritter (2007) application to symmetries on static space-time

Probability Theory By-product

See Varadhan appendix to 1968 Symanzik lectures in Varenna.

New countably-additive, probability measures on $S'(R^2)$ and on $S'(R^3)$.

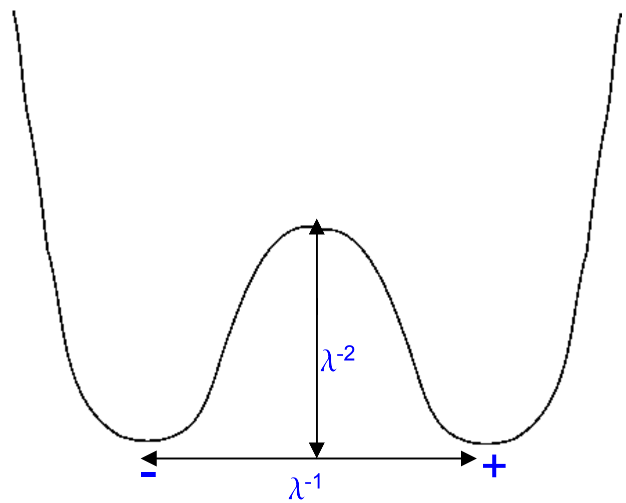
- Euclidean Invariant
- Reflection Positive
- Non-Gaussian
- Mixing (Mass Gap), Isolated particle

Glimm-J-Spencer, Guerra-Rosen-Simon, Fröhlich, Feldman-Osterwalder

Brydges, H.-T. Yau, Dimock, Hurd

Phase Transitions

Discrete symmetry breaking



$$V(\varphi) = \lambda^{-2}\left(\lambda^2\varphi^2 - 1\right)^2$$
$$\lambda \ll 1$$

Quantum Mechanics: Unique Node-less Ground State (Tunnelling)

Field Theory: Non-Unique Ground State (Possible Lack of Tunnelling)

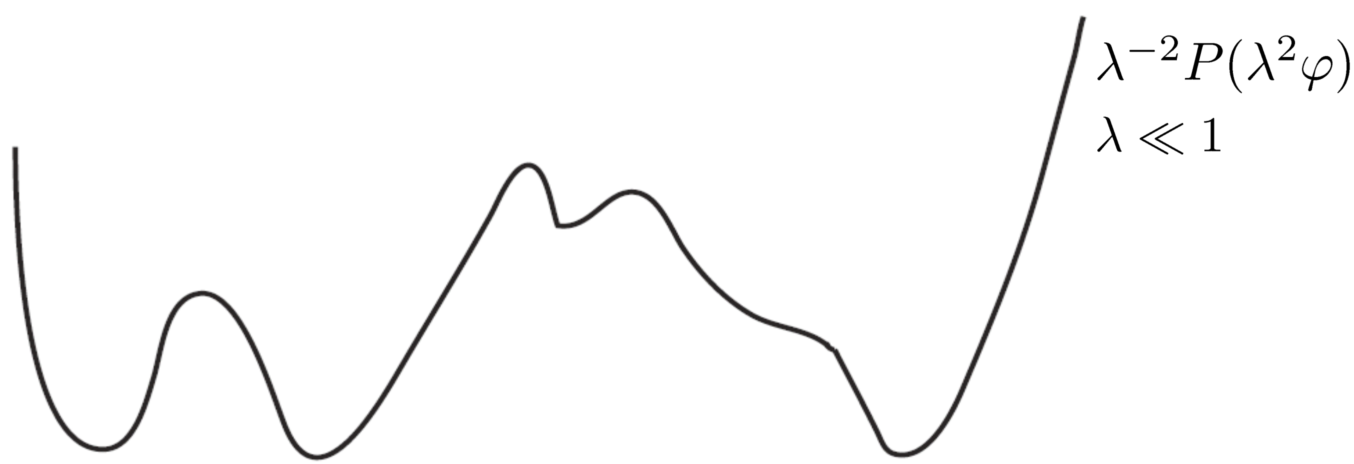
Dobrushin-Minlos: false announcement 1973, retraction 1976

Glimm-J.-Spencer: 1975, Proof in infinite volume QFT

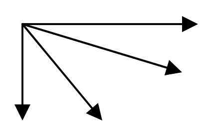
1976, m > 0 in pure phase

Interesting Playground: $\lambda^{-2}P(\lambda^2\varphi)_2$

- John Imbrie, expansion
- Piragov-Sinai picture amplified



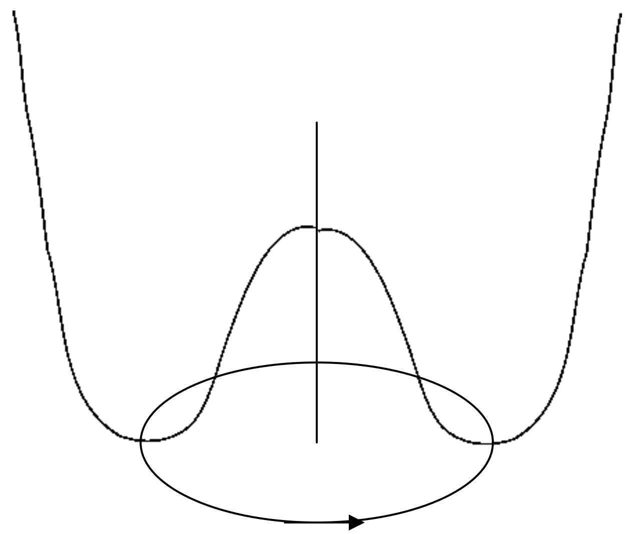
Perturb by $Q(\varphi)$ lower degree, small coefficients.
Solve integral equation for coefficients of Q.



Corner of hyper-cube

Continuous Set of Phases

Continuous Symmetry in target space



$$V(\vec{\varphi}) = \lambda\left(\vec{\varphi}^{\,2} - \tfrac{1}{\lambda}\right)^2 \;, \quad \vec{\varphi} \in R^N$$

Goldstone mode, N ≥ 2. Let $S(p) = (2\pi)^{-d/2} \sum_j e^{-ipj} \langle \varphi_0 \varphi_j \rangle$

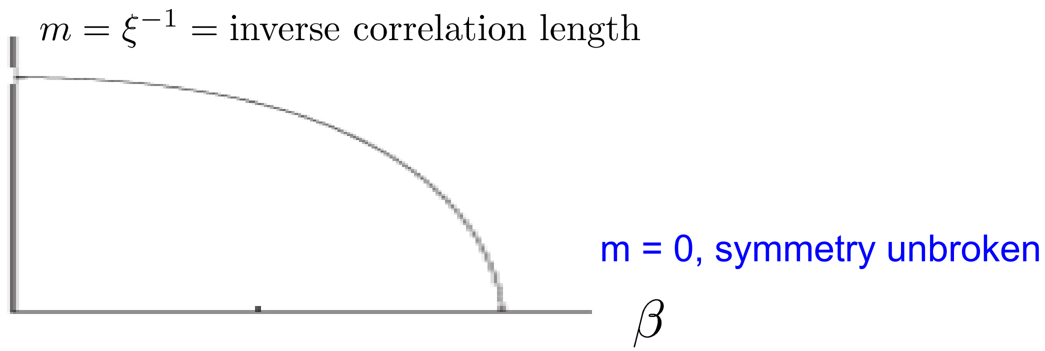
Fröhlich-Simon-Spencer (1976): Beautiful “Infra-Red Bound.” With UV cutoff,

$$0 \leq S(p) - c\delta(p) \leq c_1 \frac{1}{p^2}$$

d ≥ 3, N ≥ 1
integrable

Kosterlitz-Thouless Phase

d=2, n=2 $H = \sum (\nabla \vec{\sigma})^2$.



Fröhlich-Spencer: Tour de force, established transition.

?? d=2 symmetry unbroken. For n=3, expect m > 0, no phase transition.

Triviality of φ⁴

Dimension counting: three lines of argument

1. Sobolev Inequality holds for d ≤ 4. (PDE)

$$\epsilon \left(\int \Phi^4 dx \right)^{1/2} \leq \int \left((\nabla \Phi)^2 + \Phi(x)^2 \right) dx \ .$$

2. Asymptotic freedom holds for d < 4. (QFT)

$$\beta < 0 \ .$$

3. Random paths intersect in d < 4 (Symanzik)

$$\mathrm{Dim}_{\mathrm{H}}(\omega) = 2 \ .$$

Coupling Constant Bound

$$0 \le g = -c_d \int \langle \Phi(0)\Phi(x_1)\Phi(x_2)\Phi(x_3) \rangle^{\mathrm{T}} \, dx$$

Glimm-J. (1975) $\quad g \le M_d \; .$

Fröhlich (1982) $\quad g \le M_d \, \epsilon^{d-4} \; .$

ε is lattice spacing

Aizenman (1982)

Random Walks
Correlation Inequalities

- Symanzik
- Brydges, Fröhlich, Spencer
- Sokal
- Brydges, Fröhlich, Sokal
- Fröhlich, Felder
- Bovier, Felder
- Brydges, Imbrie

Knot Invariants

- Rigorous analysis of the Jones knot invariant, using a lattice version of Witten's field theory representation.

$$J(K) = \left\langle e^{i\int_K A} \right\rangle$$

Here the expectation is defined by an Chern-Simons action S.

Fröhlich-King (1989)

Conformal Field Theory

Conformal Theory and Representation Theory

Conformal Theory and Local Algebras

Tensor Categories

Super-symmetry

Super-symmetry

- Relation to geometry
- Super-symmetric field theories
- Twist fields provide regularization

Non-Commutative Geometry

- QFT & Entire Cyclic Cohomology
- Gauge Theory

Major Open Problem
d = 4 Field Theory

- Quantum Electrodynamics is not asymptotically free.
  - This led to the expectation that QED $\neq$ Q.E.D.

- Pure Yang-Mills theory is asymptotically free for SU(n), $n \geq 2$, suggesting it does exist.

Yang-Mills

- d=2 already mentioned.
- d=3 lattice gauge fields in finite volume Balaban, King
- Several models: Gawedzki-Kupiainen
- d = 4 breakthrough: Balaban's analysis of asymptotic freedom and large fields for lattice gauge fields
- Magnen, Rivasseau, Sénéor

- Major Problem: understand origin of mass gap.
  - Both physics and mathematics.

- Does gap arise from the quartic term $(A\wedge A)^2$ in the Yang-Mills action?

Jürg enjoys the
inexhaustible electon!

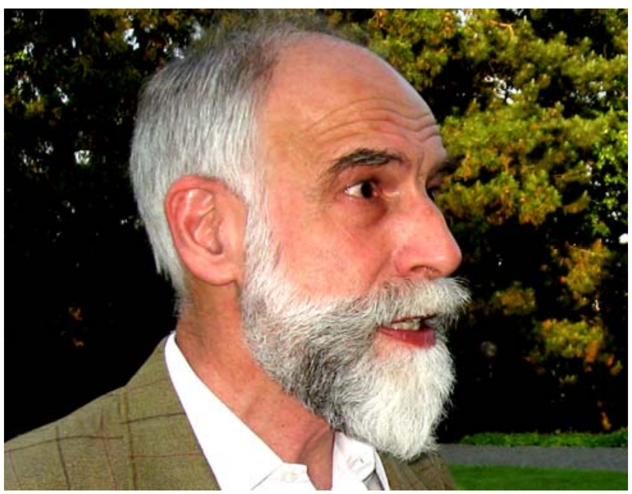
Problem for Jürg 100+

However, one can always hope for a miracle: to see a solution in my lifetime!

So lets relax!

Jürg over the Years

Les Houches 1984 / 1997



Zürich 2006-2007

Cambridge 1977

Cambridge 1977

Zürich 1996

Cargèse 1979

Hamburg 2005

HAPPY 4th of July Fireworks!!!

Dublin 2005

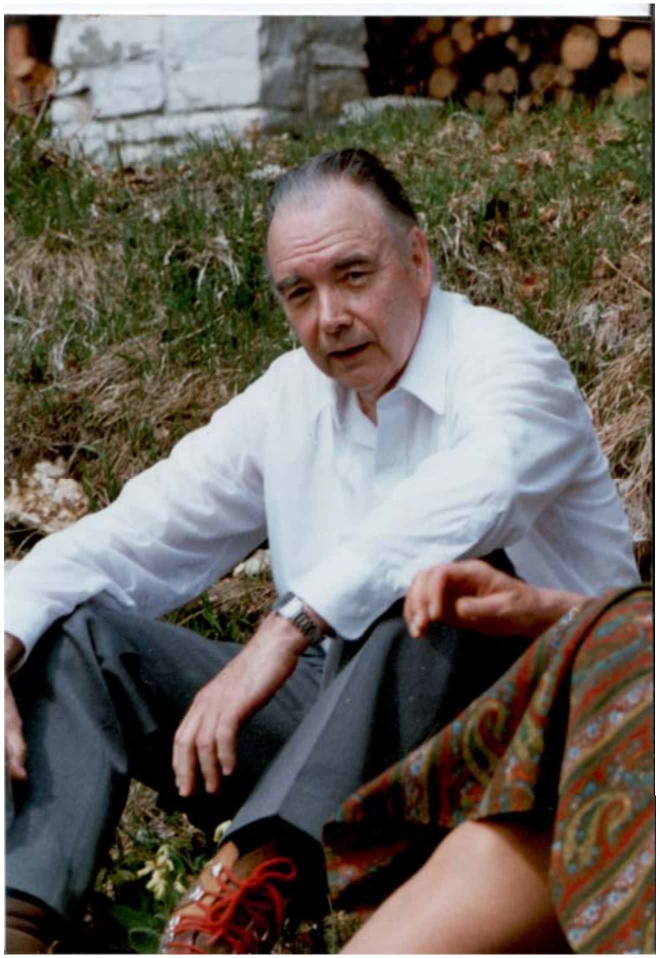
Zürich 1996

Cargèse 1979

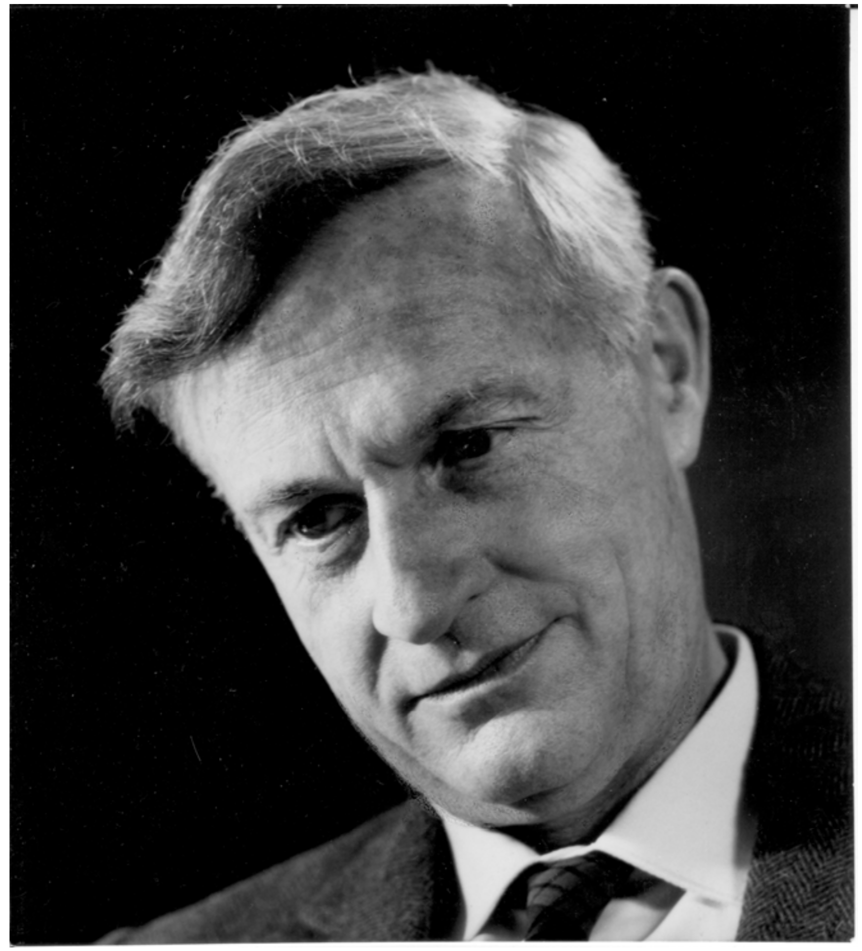
Zürich ~1992

Zürich 2006

Appendix by Klaus Hepp



Res Jost



Markus Fierz

- Vom Vater hab ich die Statur,
- Des Lebens ernstes Führen,
- Von Mütterchen die Frohnatur
- Und Lust zu fabulieren.

- Urahnherr war der Schönsten hold,
- Das spukt so hin und wieder;
- Urahnfrau liebte Schmuck und Gold,
- Das zuckt wohl durch die Glieder.

- Sind nun die Elemente nicht
- Aus dem Komplex zu trennen,
- Was ist denn an dem ganzen Wicht
- Original zu nennen?
-
- Johann Wolfgang Goethe und Deine Ahnen wünschen Dir alles Gute zum Geburtstag !